On a Multiple Relation Theory of Belief

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Abstract.

I investigate a variation of Russell's Multiple Relation Theory of Belief, which I have called a Multiple Relative Product Theory of Belief. I show how it allows for there, *truly,* being (false) beliefs about *non-existent* objects. Russell's theory of definite descriptions permits statements about *non-existent* objects to make sense, but they are always *false*. I also consider a disagreement I had with Professor Gregory Landini about the meaning of the *assertion sign* in Principia Mathematica. In addition, I discuss how it solves the problem of *opacity* which was discussed by Willard Van Orman Quine.

Abbreviations.

CPBR:	The Collected Papers of Bertrand Russell
KAKD: [1911]	"Knowledge by Acquaintance and Knowledge by Description", by Russell
ML: [1918]	Mysticism and Logic, by Russell
OD: [1905]	"On Denoting", by Russell
ONT: [1907]	"On the Nature of Truth" by Russell
ONTF: [1910]	"On the Nature of Truth and Falsehood", by Russell
OS: [1906]	"On Substitution", by Russell
PE: [1910]	Philosophical Essays, by Russell

PM: [1910*] Principia Mathematica to *56, by Whitehead and Russell

PoM: [1903] Principles of Mathematics, by Russell

PoP: [1912] The Problems of Philosophy, by Russell

TNT: [1905] The Nature of Truth, by Russell

WO: [1960] Word & Object, by Quine

WTA: [1895] "What the Tortoise said to Achilles", by Lewis Carroll

*Although I use the to *56 edition, which I expect more people have, I only use material from the 1st edition, except possibly any corrections that were made later. Page numbers are thus from the to *56 edition.

Notation.

I use some notation from PM, and some variations. I use **bold** for predicates and relations. Backward apostrophe is PM's notation for function application (*30.01). **sin**`x would, in usual algebra, be written sin(x). The relative product (*34.01) of two relations **R** and **S** is written **R|S** and is defined:

x **R|S** z =df (∃y) x **R** y & y **S** z

A relation **R** between a and b may be written either a **R** b, or **R**(a, b).

The converse of a relation is defined

x (**Cnv**`R) y =df y **R** x

I use ~ for not. I use ⊃ for material implication. Also, I use ⇔ for material equivalence. And v for disjunction (or).

I find the dot notation of PM confusing. I use & for logical product (and); I use parentheses for punctuation. I use ∀ for universal quantification, as it stands out more.

In “On Substitution”, **5a** in CPBR 5 pp. 129-232 [OS], **i** is a function from objects to ideas of the objects. **n** is a function from objects to their names.

The Assertion Sign in PM.

I am attempting to ascertain the meaning of the assertion sign “|-“ in PM. Some discussion of it occurs on page 92 of PM. Prof. Landini and I had a disagreement about this in an email discussion. He maintained that it means “*it is a theorem that*.” PM says that it may be read “*it is true that*.” It is followed by the parenthetical qualification “although philosophically this is *not exactly* what it means.” So, I will try to determine a possible more *exact* meaning.

My foremost reason for disagreeing with Landini is that it would not permit PM to justify the following reasoning:

|- Caesar died.

|- Caesar died ⊃ Caesar is dead.

therefore

|- Caesar is dead.

I believe this deduction is justified by PM *1.1, *anything implied by a true elementary proposition is true*. This is also supported by the discussion in PM, pp. 8-9. It is also supported by the discussion of Lewis Carroll’s “What the Tortoise said to Achilles”, in section 38 of PoM, which is referred to on pages 92 and 94 of PM.

None of these statements about Caesar are theorems, though I think we want to assert that they are true. And, even if they are not true, that would not make the argument invalid, only unsound. *As is a work of logic,* Whitehead and Russell *only asserted theorems*. That does not mean that they thought nothing other than theorems could be asserted. Landini often seems to *anachronistically* apply concepts developed since PM was written to PM. Another example is the concept of *well formed formulas* (wff).

Russell’s Multiple Relation Theory of Belief.

Russell considered a multiple relation theory of belief at about the time of PM. By *multiple relation*, Russell means a relation of more than 2 terms. Since most students of Russell are familiar with The Problems of Philosophy [PoP]. I will contrast the use of my terms with his example there on pp. 124-126. There Russell’s analysis is simpler but has problems that I will explain. Russell’s analysis in PoP involves a relation *believes*. In his example it is used to illustrate a case with 4 terms from Shakespeare’s tragedy Othello:

believes(Othello, loves, Desdemona, Cassio)

In this PoP version of Russell’s analysis, if Cassio did not exist, the proposition that **believes**(Othello, loves, Desdemona, Cassio) is nonsense. Russell, at least officially at that time, believed the terms of a propositions were the objects in the world. So, if a term had no reference, then a sentence containing it was nonsense.

Russell’s Theory of Definite Descriptions.

Russell did have what was a partial solution for this, his analysis of definite descriptions first proposed in “On Denoting” in **16** CPBR 4, pp. 414-427 [OD]. It was formalized in PM.

There are 4 definitions there, here is a simplified version of the first.

$$\boldsymbol{\psi}(\iota x)(\boldsymbol{\phi} x) =df (\exists b)(\forall x)(\boldsymbol{\phi} x \Leftrightarrow . x=b) \;\&\; \boldsymbol{\psi} b$$

With this analysis, a meaning is given for propositions involving “the so and so”, when there is no such entity, but *the propositions are always false*. Namely, in this example it would be “There is one and only one x such that **f**(x) if and only if x=b and **believes**(Othello, loves, Desdemona, b)”, where **f** describes Cassio. However, this is false if there is no such x. *So, in this analysis, there can not truly be beliefs about non-existent objects,* although they are not nonsense. Any statement of such a belief would be false.

Other descriptions of belief found in Russell.

Russell's position was essentially the same in "On the Nature of Truth and Falsehood" [ONTF] published in Philosophical Essays [PE] and as **12** in CPBR, Vol 6, pp. 115-124.

Russell proposed a different version of his analysis in section III of "On the Nature of Truth" ONT, published in 1907 in the Proceedings of the Aristotelean Society, n.s. 7. Russell did not permit this section to be reprinted when the paper was reprinted in PE. The whole paper is **14** in CPBR, Vol 5, pp. 433-454. In it, belief is a relation of ideas in a person's mind, instead of objects. However, there is *no description of a relation of ideas to objects given*, although such a relation is implied.

In Russell's work 1906 work notes OS published in as **5a** CPBR, Vol 5 he developed a theory, on pp. 185ff., almost identical to mine. Instead of taking "A **believes** $\boldsymbol{\psi}$(a, b)" to indicate a dyadic relation of belief between person A and an entity $\boldsymbol{\psi}$(a, b), we find that Russell considered a relation of ideas $\boldsymbol{\Psi_i}$ that holds between **i**`a which is an idea A has of a and **i**`b which is an idea A has of b. Similarly "A **asserts** that $\boldsymbol{\psi}$(a, b)" is to be taken as involving a relation $\boldsymbol{\Psi_n}$ that holds between **n**`a which is a name A has of a and **n**`b which is a name A has of b. Belief involves ideas before the mind, while assertions involved names (or words).

Definitions for my multiple relation theory.

I am attracted to an approach Russell experimented with in OS, pp. 185ff. and had independently developed something similar. I was pleased to see that Russell had anticipated these ideas in his work notes.

On this theory in my terminology, for a person A at time t, *there is a relation **S** between the person's ideas and objects corresponding to the ideas*. *There is also another relation* **R** *between names (words) and the corresponding ideas*. These relations may be analyzable, I am making no claim about whether or not they can be further analyzed. We will have:

$\mathbf{R}^{\mathbf{A}}$(x, y) =df x is an name A has of idea y.

$\mathbf{S}^{\mathbf{A}}$(y, z) =df y is an idea A has of object z.

These can also be written

x $\mathbf{R}^{\mathbf{A}}$ y

y $\mathbf{S}^{\mathbf{A}}$ z

and (in this case)

x $\mathbf{R}^{\mathbf{A}}|\mathbf{S}^{\mathbf{A}}$ z

x is a word for the object z (for A)

If we add a temporal component t, we can write:

$\mathbf{R}^{\mathbf{A,t}}$(x, y) =df at time t, x is an name A has of idea y.

$\mathbf{S}^{\mathbf{A,t}}$(y, z) =df at time t, y is an idea A has of object z.

These can also be written

x $\mathbf{R}^{\mathbf{A,t}}$ y

y $\mathbf{S}^{\mathbf{A,t}}$ z

and (in this case)

x $\mathbf{R}^{\mathbf{A,t}}|\mathbf{S}^{\mathbf{A,t}}$ z (for A at time t)

When not needed, I will not specify A and t.

I introduce a new special case of definite descriptions - *14.05.

$((\iota\boldsymbol{\psi})(\mathbf{S}(\boldsymbol{\phi}, \boldsymbol{\psi})))\ (\ ((\iota c)(\mathbf{S}(a, c)), ((\iota d)(\mathbf{S}(b, d))\ ) =\mathrm{df}$

$(\ (\exists\boldsymbol{\psi})\ (\iota\mathbf{f})\ (\mathbf{S}(\boldsymbol{\phi}, \mathbf{f})) = \boldsymbol{\psi})\ \&$

$(\ (\exists c)\ (\iota u)\ (\mathbf{S}(a, u)) = c)\ \&$

$(\ (\exists d)\ (\iota v)\ (\mathbf{S}(b, v)) = d)\ \&$

$\boldsymbol{\psi}(c, d)$

Note, $\boldsymbol{\phi}$**,** a, and b are constants (ideas) here. Three entities described are $\boldsymbol{\psi}$**,** c, and d. $\boldsymbol{\psi}$ is the relational object such that $\boldsymbol{\phi}$ is uniquely related to by the S relation. Please notice that this $(\iota\boldsymbol{\psi})(\mathbf{S}(\boldsymbol{\phi}, \boldsymbol{\psi})$ is a definite description in predicate position. I believe this was not done in <u>PM</u>, but it is defined in terms of which <u>PM</u> does use with predicates – quantification and identity.

My primary belief relation.

For me, belief is primarily a relation of a person to several of the person's ideas – $\mathbf{B}_{\mathbf{primary}}$(A, f, a, b). Here, A is a person. f is the idea A has of a predicate or relation. a and b are other ideas. There can be various arites, and a time could also be specified, e.g. $\mathbf{B}_{\mathbf{primary}}$(A, t, a, b, c, d). If $(\exists x)S(a, x)$, then there is an object a is the idea of. Generally, we only have direct access to our ideas. We infer the existence of objects to make sense of regularities of our ideas. We also, in that process infer the existence of the R and S relations. That is the source of intentionality. Ordinarily, the R and S relations are unconscious, but they can be consciously inferred. External objects only, here, occur as values of variables. In some cases, there may not be such an object:

$\sim(\exists z)S(a, z)$

Also, we may have ideas for which we have no words:

$\sim(\exists x)\mathrm{R}(x, a)$

If we have both a word and object, then:

$(\exists x, z)\ x\ \mathrm{R|S}\ z$

In addition, there may be objects that we have no idea of:

$(\exists z)\sim(\exists y)\mathrm{S}(y, z)$

Or objects which are not words for any idea.

$(\exists x)\sim(\exists y)\mathrm{R}(x, y)$

Words and ideas are special cases of objects.

In addition to **R** and **S**, I take there to be a psychological relation, possibly analyzable if we learn more about psychology, that may hold between a person's ideas – even if no corresponding objects exist. I call it $\mathbf{B}_{\mathbf{primary}}$. There may be objects for which a person has no ideas, but in such a case the person could not form singular beliefs about those specific objects. Also, there may be objects for which a person has no words and thus cannot make singular assertions about those objects.

My Relations defined in terms of Russell's Relations.

Russell used two relations **i** and **n** in OS. My **R** and **S** can be defined in terms of those as:

$x\ \mathbf{R}\ a =df\ (\exists z)\ ((a = \mathbf{i}`z)\ \&\ (x = \mathbf{n}`z))$

$a\ \mathbf{S}\ z =df\ (a = \mathbf{i}`z)$

Here a is an idea, which is primary for me. The main difference between Russell and me is that he takes objects as definitive. Then there can be no definition of ideas or names for which there is no object.

Russell's Relations defined in terms of my Relations.

Defining Russell's relations in terms of **R** and **S**:

n`b = x =df (∃y) (x **R** y) & (y **S** b)

i`b = y =df (y **S** b)

Here b is an object. This amounts to:

n =df **Cnv` (R|S)**

i =df **Cnv`S**

As I take ideas as primitive, there can be beliefs about ideas for which no object exists. As Russell takes objects as primitive, he cannot describe ideas with no corresponding object.

The Intentionality of $\mathbf{Belief_{full}}$ and $\mathbf{Belief_{private}}$

$\mathbf{Belief_{primary}}$, **R**, and **S** are primitives here. I do not exclude the possibility of further analysis, especially from psychology.

A $\mathbf{believes_{full}}$ that ϕ(a, b) =df

(∃f, x, y) $B_{primary}$(A, f, x, y) & S^A(f, ϕ) & S^A(x, a) & S^A(y, b)

This $\mathbf{belief_{full}}$ is about objects – not ideas. This is the source of *intentionality*. Ideas are *about* objects.

This is close to Russell's analysis in PoP. It is never true if any of ϕ, a, or b do not exist. In fact, it would be nonsense unless such a ϕ, a, or b were given as a description. I am regarding this $\mathbf{belief_{full}}$ to be judged by an independent third party.

$\mathbf{Belief_{private}}$ is all a person can know about his own belief.

A $\mathbf{believes_{private}}$ that **f**(x, y) =df

$B_{primary}$(A, f, x, y) & S^A(f, (ιϕ)(S^A(f, ϕ)**)** & S^A(x, (ιa)(S^A(x, a)**)** & S^A(y, (ιb)(S^A(y, b)**)**

As f, x, and y are ideas in A's mind they are guaranteed to exist. **Belief**$_{private}$ declares the *intentionality* of belief. It makes sense even if there are no such ϕ, a or b, but in such cases it is false. There is no *aboutness* of *non-existent* objects, but it will be felt as if there were such aboutness.

True belief$_{full}$ and True belief$_{private}$

A **believes_truly**$_{full}$ that $\phi(a, b)$ =df

$(\exists f, x, y)\ B_{primary}(A, f, x, y)\ \&\ S^A(f, \phi)\ \&\ S^A(x, a)\ \&\ S^A(y, b)\ \&\ f(x, y)$

As this directly involves objects, if such objects do not exist, it is nonsense, but person A cannot tell. He has the ideas. One cannot tell from an idea, whether the corresponding object exists

A **believes_truly**$_{private}$ that $\mathbf{f}(x, y)$ =df

$B_{primary}(A, f, x, y)\ \&\ (\iota\phi)(S^A(f, \phi))\ ((\iota a)(S^A(x, a)),\ (\iota b)(S^A(y, b)))$

Here I am using the use of definite descriptions that I introduced in *14.05. It only directly involves ideas. It makes sense even if the objects do not exist, but then it is false.

False belief$_{full}$ and False belief$_{private}$

A **believes_falsely**$_{full}$ that $\phi(a, b)$ =df

$(\exists f, x, y)\ B_{primary}(A, f, x, y)\ \&\ S^A(f, \phi)\ \&\ S^A(x, a)\ \&\ S^A(y, b)\ \&\ \sim f(x, y)$

As this directly involves objects, if such objects do not exist, it is nonsense, but person A cannot tell. He has the ideas. One cannot tell from an idea, whether the corresponding object exists

A **believes_falsely**$_{private}$ that $\mathbf{f}(x, y)$ =df

$B_{primary}(A, f, x, y)\ \&\ \sim(\iota\phi)(S^A(f, \phi))\ ((\iota a)(S^A(x, a)),\ (\iota b)(S^A(y, b)))$

Here I am using the use of definite descriptions that I introduced in *14.05. It only directly involves ideas. It makes sense even if the objects do not exist.

Assertion defined.

We are now ready to define assertion, which is distinct from belief. I take **says** as a primitive relation between a person and names (i.e. words).

$|\!-^{A}\ \boldsymbol{\Phi_n}(a, b)$ =df

i.e.

A asserts that $\boldsymbol{\Phi_n}(a, b)$ =df

$(\exists f, x, y)$ says$(A, \boldsymbol{\Phi_n}, a, b)$ & $\mathbf{R}^{\mathbf{A}}(\boldsymbol{\Phi_n}, f)$ & $\mathbf{R}^{\mathbf{A}}(a, x)$ & $\mathbf{R}^{\mathbf{A}}(b, y)$

$\boldsymbol{\Phi_n}$, a, and b are words in the assertion, so they exist if person A made the assertion (said it). There will be f, x, and y if the words have meaning to person A. If no such (ideas) f, x, or y then the existential quantifier fails and there is no assertion. Also, perhaps, the ideas must be limited to ones which make sense when so combined. That is beyond the scope of this paper and will not be considered. There may be no objects (corresponding to the ideas) with **Cnv`S** relations to f, x, or y. In that case, there can still be such an assertion. That is A may use words which correspond to ideas, but ideas to which no objects correspond. Yet, an assertion is made. Russell had a problem in this case as, for him, the words are described in terms of objects. He has no relation between words and ideas, if there is no object. The **R** relation that I am using does not require there to be such an object. We may have both words and ideas to which no object corresponds. We are considering that there may be no such objects.

True Assertion.

$|\!-^{A}{}_{true}\ \boldsymbol{\Phi_n}(a, b)$ =df i.e.

A asserts truly that $\mathbf{\Phi_n}(a, b)$ =df

$(\exists f, x, y)$ **says**$(A, \mathbf{\Phi_n}, a, b)$ & $\mathbf{R^A}(\mathbf{\Phi_n}, f)$ & $\mathbf{R^A}(a,x)$ & $\mathbf{R^A}(b, y)$ &

$(\iota\phi)(S^A(f, \phi))$ **(**$(\iota u)(S^A(x, u)$**)),** $(\iota v)(S^A(y, v)$**)))**

Here, again, I am using *14.05.

This is different. To be true, there must be objects with the Cnv`S relations to the ideas f, x, and y. We also need $(\iota\phi)(S^A(f, \phi)$**) (**$(\iota a)(S^A(x, a)$**)),** $(\iota b)(S^A(y, b)$**))).** This is defined by *14.05.

The objects are specified by definite descriptions, so the assertion makes sense (although false) if the objects do not exist. Person A does not assert truly if any of the objects do not exist, but the person does assert by the previous definition. There cannot be a true assertion if any of the objects described do not exist. If any of the ideas do not exist, then there is no assertion (even false). In that case, some of A's words have no meaning.

Note you can assert things that you do not believe. Honest assertion is:

$\vdash^{A}_{honestly}$ $\mathbf{\Phi_n}(a, b)$ =df

i.e.

A asserts honestly that $\mathbf{\Phi_n}(a, b)$ =df

$(\exists f, x, y)$ **says**$(A, \mathbf{\Phi_n}, a, b)$ & $\mathbf{R^A}(\mathbf{\Phi_n}, f)$ & $\mathbf{R^A}(a,x)$ & $\mathbf{R^A}(b, y)$ &

$\mathbf{belief_{primary}}(f, x, y)$

False Assertion.

A person asserts falsely if they assert but do not assert truly. Asserting falsely is not the same as it being false that one asserts.

$|\!-\ ^{A}{}_{false}\ \mathbf{\Phi_n}(a, b) =df$

i.e.

A asserts falsely that $\mathbf{\Phi_n}(a, b)$ =df

$(\exists f, x, y)$ **says**$(A, \mathbf{\Phi_n}, a, b)$ & $\mathbf{R}^A(\mathbf{\Phi_n}, f)$ & $\mathbf{R}^A(a,x)$ & $\mathbf{R}^A(b, y)$ &

$\sim(\iota\phi)(S^A(f, \phi))\ ((\iota u)(S^A(x, u)), (\iota v)(S^A(y, v)))$

Here, again, I am using *14.05.

This requires A to make the statement and to have ideas of A's words. Otherwise it is not even a false assertion. The assertion is false if there are not objects corresponding to any of the ideas or the corresponding relation does not hold among them.

Russell's Objections to Ideas.

In the Essay "Knowledge by Acquaintance and Knowledge by Description" – in Mysticism and Logic [ML] and **15** CPBR 6, pp. 147-161. [KAKD], of which a shorter version was in PoP, Russell objected to ideas. He says, "The view seems to be that there is some mental existent which may be called the 'idea' of something outside the mind of the person who has the idea, and that, since judgement is a mental event, its constituents must be constituents of the mind of the person judging. But in this view ideas become a veil between us and outside things – we never really, in knowledge, attain to the things we are supposed to be knowing about, but only to the ideas of those things. The relation of mind, idea, and object, on this view, is utterly obscure, and, so far as I can see, nothing discoverable by inspection warrants the intrusion of the idea between the mind and the object." (pp. 221-222 ML).

First, I want to note what Russell states to be his *fundamental epistemological principle*. Namely, "*every proposition which we can* *understand must be composed wholly of constituents with which we are acquainted.*" (p. 219).

Next, I want to note what he takes us to be acquainted with. "There are thus at least two sorts of objects of which we are aware [acquainted with], namely, particulars and universals. Among particulars I include all existents, and all complexes of which one or more constituents are existents, such as this-before-that, this-above-that, the-yellowness-of-this. Among universals I include all objects of which no particular is a constituent." (pp. 213-214) He also says, "Awareness of universals is called *conceiving*, and a universal of which we are aware is called a *concept*." (p. 212)

Next, note that "it will be seen that among the objects with which we are acquainted are not included physical objects (as opposed to sense-data), nor other people's minds. These things are known to us by what I call 'knowledge by description, …]'" (p. 214)

I think my *ideas* are not a barrier between the mind and world any more than Russell's *concepts*. But any statement of belief, using a description of a non-existent particular, will be false – using Russell's analysis of definite descriptions, e.g. children could not believe that Santa lives at the north pole. On Russell's analysis it would have to be false, though not nonsense. On my view, children can $\textbf{believe}_{\textbf{primary}}$ it. The $\textbf{believe}_{\textbf{primary}}$ could be true (true they believe it), though a false belief. $\textbf{believes}_{\textbf{full}}$ would be nonsense. $\textbf{believes}_{\textbf{private}}$ would make sense, but it be false. Also, it could be asserted, but it would be a false assertion.

On Denoting.

Russell gave three tests in <u>OD</u>, that a theory of denoting should pass. (**16** in CPBR, Vol 4, p. 420).

(1) "If a is identical with b, whatever is true of the one is true of the other, and either may be substituted for the other in any proposition without altering the truth or falsehood of that proposition." Russell's example is "George IV wished to know whether Scott was the author of *Waverley*." On my analysis wished_to_know is a propositional attitude analogous to $\text{believes}_{\text{private.}}$ **(GeorgeIV, identical, Scott, the_author_of_Waverley) =df** $B_{\mathbf{primary}}$(A, idea_of_∃, idea_of_x, idea_of_∀, idea_of_y, idea_of_wrote_Waverley, idea_of_y, idea_of_⇔, idea_of_x, idea_of_identity, idea_of_Scott)

& $\mathbf{S}^{A}$(idea_of_Scott, (ιa)(S^{A}(idea_of_Scott, a**))**

& $\mathbf{S}^{A}$(idea_of_wrote_Waverley,(ιb)(S^{A}(idea_of_wrote_Waverley, b)**)**

I would have preferred to use Polish notation in a case such as the above. I omitted punctuation which the reader must supply; it is not needed in Polish notation.

I think one must admit some ideas for which no object corresponds, such as Santa. Otherwise, children could not believe Santa lived at the north pole. If Santa were analyzed into a description in terms of things with which we are acquainted, and if there were no such object, it would be false, on a multiple relation theory, that such a belief existed. There can truly be assertions about non-existent objects as well, on my analysis, though they will always be false.

However, I would not have to admit ideas such as of round squares, I could analyze them into simple ideas.

(2) The law of the excluded middle.
The definitions are based on $\mathbf{B}_{\mathbf{primary}}$ to which the law of excluded middle would apply. And it would also apply to **R** and

S. Cases like the round square would be handed as Russell did, as would the present king of France being bald.

(3) What is interesting here is *meaning* vs *denotation*. On my theory, the domain of **S**, and the converse domain of **R** are ideas. The domain of **R** is words, and the converse domain of **S** is objects. Usually, ideas are meanings, but all ideas are themselves objects, so we can have ideas of ideas. Also, all words are objects. We can have ideas about words, and consequently words about words. This type of thinking is common in computer programming. The converse domain of **S** is to be thought of as denotation.

Opacity of Belief.

Next, I discuss a case of Quine's given in Word & Object [WO], pp. 142-146.

Remember,

A $\mathbf{believes}_{\mathbf{private}}$ that $\mathbf{f}(x, y)$ =df

$B_{primary}(A, f, x, y)$ & $\mathbf{S}^A(f, (\iota\phi)(S^A(f, \phi)))$ & $\mathbf{S}^A(x, (\iota a)(S^A(x, a)))$ & $\mathbf{S}^A(y, (\iota b)(S^A(y, b)))$

Let

A = Tom

f = Tom's idea of denouncing

x = Tom's idea of Cicero

y = Tom's idea of Catiline

z = Tom's idea of Tully

ϕ = denouncing

a = Cicero

b = Catiline

c = Tully

Now Tom believes Cicero denounced Catiline

but Tom also believes Tully did not denounce Catiline,

even though Cicero = Tully. (a = c)

Tom does not know of this identity.

We have $(\exists x, z)\ \mathbf{S}^A(x, a)\ \&\ \mathbf{S}^A(z, a)$

$\mathbf{B}_{primary}(A, f, x, y)\ \&\ \mathbf{S}^A(f, (\iota\phi)(S^A(f, \boldsymbol{\phi})\mathbf{)})\ \&\ \mathbf{S}^A(x, (\iota a)(S^A(x, a)\mathbf{)})\ \&\ \mathbf{S}^A(y, (\iota b)(S^A(y, b)\mathbf{)})$

and

$\mathbf{B}_{primary}(A, f, z, y)\ \&\ \mathbf{S}^A(f, (\iota\phi)(S^A(f, \boldsymbol{\phi})\mathbf{)})\ \&\ \mathbf{S}^A(z, (\iota a)(S^A(z, a)\mathbf{)})\ \&\ \mathbf{S}^A(y, (\iota b)(S^A(y, b)\mathbf{)})$

These are not equivalent even though $(\iota a)(S^A(x, a)) = (\iota a)(S^A(z, a))$

because the x and z still occur in $\mathbf{B}_{\mathbf{primary}}$, and $x \sim= z$.

x and z, here are constant ideas in A’s mind (not variables).

So $\mathbf{B}_{primary}(A, f, x, y) \sim\Leftrightarrow \mathbf{B}_{primary}(A, f, z, y)$

And consequently:

A $\mathbf{believes}_{\mathbf{private}}$ that $\mathbf{f}(x, y) \sim\Leftrightarrow$ A $\mathbf{believes}_{\mathbf{private}}$ that $\mathbf{f}(z, y)$,

Although $\mathbf{S}^A(x, a)\ \&\ \mathbf{S}^A(z, c)$, where a = c.

Intension vs. Extension.

I think I find this suggestive of an additional relation that I will call **T**. **T** is a many-many relation analogous to the many-one relation **S**. For example:

idea_of_humans **T** any member of the class of humans

idea_of_featherless_bipeds **T** any member of the class of featherless bipeds

where

class of humans = class of featherless bipeds

but

idea_of_humans ~= idea_of_featherless_bipeds

I have just realized this possibility. I had always accepted PM's definition of [extensional] classes contextually in terms of [intensional] relations. I no longer accept it. I think logic itself deals with classes fundamentally. One problem was that it seems there must be many more classes that one can believe there are intensions. E.g. there are a non-denumerable number of real numbers. But it is difficult for me to believe that there are a non-denumerable number of intensions. Also, in some cases there could be multiple intensions with the same extension, as in the case of featherless bipeds and humans. We can, however, describe intensions in extensional terms. We can presumably often know facts about intensions in some cases. Most classes are not describable. Facts about intensions are facts of psychology, but still, I think, of interest to philosophical logic, because of confusion they can cause. I think pure logic needs only individuals, classes, and some logical notions. We can however only know these through our ideas.

Conclusion.

The theory presented here solves the problems associated with false belief and assertion, especially in the cases of non-existent entities.

Russell believed <u>OD</u> solved these problems. However, <u>OD</u> requires a primitive predicate for any such entity. The theory I propose only requires the **R**, **S**, **T**, and $\mathbf{B}_{\mathbf{primary}}$ relations in addition to the usual logical notions. The apparent opacity of belief is also resolved in a simple case. We are only able to think in ideas. We posit entities to correspond to our ideas, but only know them by description. The only real things that we directly know are our ideas and experiences. It seems we also have some innate logical ideas, although we develop those through experience. Psychology follows the laws of logic, as any science does. But people need not think logically.

I think it appropriate to quote Russell's concluding paragraph from <u>OD</u> – **14** CPBR, Vol 4, p. 427, "Of the many other consequences of the view I have been advocating, I will say nothing. I will only beg the reader not to make up his mind against the view – as he might be tempted to do, on account of excessive complication – until he has attempted to construct a theory of his own on the subject of denotation. This attempt, I believe, will convince him that, whatever the true theory may be, it cannot have such a simplicity as one might have expected beforehand."

Future Work.

More needs to be done on the ideas corresponding to logical notions, especially variables and quantifiers. Opacity would also need to be examined in such cases. I need to explain the beliefs of one person about another person's beliefs. I also need to consider the nature of propositions, and synonymy. How can two people have the same belief? Is it an equivalence class of $\mathbf{belief}_{\mathbf{primary}}$ relating to the same objects or classes via **S**, or **T**? Also, the nature of scientific laws needs to be explored. I also need a relation such as **S** or **T** for dual or multiple relations – or to somehow modify **T** for such cases.