An Argument for Completely General Facts: Generalized Molecular Formulas in Logical Atomism

Abstract

In his 1918 logical atomism lectures, Russell argued that there are no molecular facts. But he posed a problem for anyone wanting to avoid molecular facts: we need truth-makers for generalizations of molecular formulas, but such truth-makers seem to be both unavoidable and to have an abominably molecular character. We might call this the *problem of generalized molecular formulas*. I clarify the problem here by first distinguishing two cases: *incompletely generalized* molecular formulas and *completely generalized* molecular formulas. I next argue that, if empty worlds are logically possible, then the model-theoretic truth-functional considerations that are usually given address the first kind of formula, but not the second kind. I then show that the commitments Russell has already made provide an answer to the problem of completely generalized molecular formulas. An upshot of this discussion is that, assuming empty worlds are logically possible, some truth-makers will be general facts that have no constituents.

Keywords *logical atomism*, *truth maker*, *metaphysical nihilism*, *fact*, *empty worlds*

Russell argued in his 1918 logical atomism lectures that there are no molecular facts, though he held that there are positive and negative atomic facts, and that there are universal and existential facts. However, he posed a problem for anyone wanting to avoid molecular facts: we need truth-makers for generalizations of molecular formulas—formulas such that a molecular connective occurs in the scope of a quantifier. But such truth-makers seem to be have an abominably molecular character because some of the formulas that they make true seem ineliminably molecular. I call this the *problem of generalized molecular formulas*.

Most scholars working in metaphysics today agree that molecular truth-makers are to be avoided. They usually avoid molecular truth-makers by appealing to truth-functional considerations: once we have truth-makers for Fa and Gb, we can just say that what makes $Fa \wedge Gb$ true is the non-molecular facts Fa and Gb.

I argue here that such considerations do not solve the problem because they do not work for generalized molecular formulas. First, I distinguish two very different sorts of generalized molecular formula. The first are what we might call material claims involving some non-variable terms, like 'all humans are mammals'. The second are what we might call logical claims involving only variable terms. These include formulas of logic, such as

$$\forall x[x = x \rightarrow x = x], \qquad \forall x \forall F[Fx \rightarrow Fx], \qquad \exists x \exists F[Fx \rightarrow Fx], \qquad \text{etc.}$$

I then argue that, while the standard truth-functional considerations do apply to the first sort of generalized molecular formula, they cannot, under some plausible assumptions, account for the second sort. The assumptions are (a) an empty world is logically possible, (b) logic, if applicable to an empty world, is not *trivial* in the sense that some but not all formulas are true, and (c) we are not *ante rem* realists, or 'platonists', about universals.

After showing how the standard arguments against molecular facts fall flat for such formulas, I give a proposed solution the problem. I show how to avoid molecular facts by positing completely generalized facts instead. These completely generalized facts will not be mereological sums of entities, but *sui generis* entities that may exist even if nothing concrete existed in the universe.

Besides solving the problem of generalized molecular formulas, there are two notable upshots of what follows. First, we will see that, if the assumptions (a), (b), and (c) are accepted, then we are committed to non-atomic truth-makers, and some of these will seemingly have no constituents. Second, we will see that the common view, presented in §1, that logical atomism is partly defined by the *principle of atomicity*, on which only atomic facts are truth-makers, is implausible on metaphysical and historical grounds.

Rejecting the principle of atomicity is not novel: past philosophers, like Russell and Armstrong, and present ones, like Barker and Jago, have previously argued for non-atomic truth-makers.[1] The novel part of the discussion here, and of reconsidering the problem of molecular formulas raised in Russell's 1918 logical atomism lectures, is to raise deep difficulties in the current accounts of non-atomic facts as compositions, mereological or otherwise, from atomic facts.[2] Chief among these difficulties, as the argument below will show, is that some non-atomic facts have no constituents, and so are neither composed of anything, nor even compositional, at all.

1 Logical Atomism and the Principle of Atomicity

Some scholars characterize logical atomism as a view about truth-makers.[3] Modern truth-maker theory is anticipated in Russell's 1918 logical atomism lectures and in Wittgenstein's 1921 *Tractatus*.[4] *Truth-makers*

[1] Armstrong defended general facts in (Armstrong, 2004, 74) and (Armstrong, 1997, 135). Russell defends general and negative facts (Russell, 1918/1986, 190, 207). Russell's defense of negative facts is highly tentative (Perovic, 2018, §2). Not to be outdone by the dead, Barker and Jago defend general, molecular, and negative facts (Barker and Jago, 2012, 125-126).

[2] A good discussion of non-mereological composition is in (Barker and Jago, 2012, §3).

[3] "Logical atomism is a view about the relationship between truths and what make truths true...If P is a class of propositions, logical atomism with respect to P is the view that all the true propositions in P are made true by atomic facts." (Simons, 1992, 158, 160)

[4] "When I speak of a fact...I mean the kind of thing that makes a proposition true or false." (Russell, 1918/1986, 163) Wittgenstein seems to view facts as truth-makers without describing them in those words (Wittgenstein, 1922/1971, 2.222, 4.063, 5.101). The notion of correspondence occurs in his early writings, too, including his 1913 *Notes on Logic* (Wittgenstein, 1913/2009, B10).

are the really existing stuff that makes formulas true, or in virtue of which formulas are true, when they are true.[5] Note that truth-maker theory, so characterized—and this characterization is standard—stands or falls with an account of the truth-making relation, that is, of what making true consists in.[6] However, truth-maker theorists differ in their account of it (MacBride, 2016, §1). For present purposes, we can take truth-making as consisting in something like correspondence to how the world is.[7]

Unfortunately, it is quite difficult to determine what really existing stuff is needed to make formulas true. Let us take a true affirmative claim, like *Bertrand Russell is human*. For the sake of argument, let us suppose that it is an atomic fact that makes this true, that is, an atomic fact Hb, which is an instance of the general form $R^n(a_1, ..., a_n)$. Atomic facts consist of some property holding of one or more things. They are, as Russell says, "as facts go very simple" (Russell, 1918/1986, 177). Most everyone who buys into truth-maker theory and an ontology of facts, or states of affairs, posits such really existing stuff like atomic facts.

Now take any negative true claim, like *Bertrand Russell is not alive*, which for the sake of argument we will suppose is similarly made true by an atomic fact. But now truth-maker theorists have a problem. This fact appears to have a rather different form from whatever is a truth-maker for a positive true claim, for it is precisely the property of *being alive* failing to hold of Russell that makes it the case that he is not alive. Yet this places us in the uncomfortable position of saying that the really existing stuff, which makes it true that *Bertrand Russell is not alive*, is somehow not existing, uncombined, or combined differently.

Enter logical atomism. According to some truth-maker theorists, *logical atomism* is the view that only atomic facts are truth-makers. Different scholars assess this thesis with quite varying degrees of enthusiasm, but a fair number of them seem united in viewing logical atomism as the thesis in truth-maker theory that only atomic facts are truth-makers:

> The glory of logical atomism was that it showed that not every kind of sentence needs its own characteristic truth-maker. Provided we can account for the truth and falsehood of atomic sentences, we can dispense with special truth-makers for, e.g., negative, conjunctive, disjunctive, and identity sentences. (Mulligan et al., 1984, 289)
>
> The idea that the complex natural world is reducible to ontologically simple objects and atomic states of affairs is a difficult, if not impossible, thesis to defend. (Cocchiarella, 2007, 141)
>
> Logical atomism is designed to go with the ontological view that the world is the totality of atomic facts...doing without funny facts: atomic facts are all the facts there are...Logical complexity, so

[5] "The idea of a truthmaker for a particular truth, then, is just some existent, some portion of reality, in virtue of which that truth is true." (Armstrong, 2004, 5)

[6] "Truthmaker theory is a theory about *what it is* for a proposition to be true; it's just not the kind of theory that can apply only to a restricted domain." (Cameron, 2008, 412)

[7] Truth-maker theory can be viewed as a species of the *correspondence theory of truth* on which a formula being true consists in its correspondence to some feature of the world. The truth-maker theorist would add that this correspondence consists in truth-making. Confer (David, 2016, §8.5) and (MacBride, 2016, 3.3).

> the idea goes, belongs to the structure of language and/or thought; it is not a feature of the world. (David, 2016, §7.1)

Following Russell, who himself attributes a version of it to Wittgenstein, I call this claim the 'principle of atomicity': the *principle of atomicity* is the claim that only atomic facts are truth-makers.[8] Note that not all truth-bearers are atomic: the *atomistic hierarchy of sentences*, as Russell calls it, is the collection of all formulas closed under substitution, truth-functional combination, and generalization.[9] It is the collection of all formulas that can be constructed out of elementary formulas from these specific operations.[10] The chief point is that only atomic facts are truth-makers, even though not all formulas are atomic. So an upshot of the principle of atomicity is a sparse ontology: the world consists of atomic facts alone and their constituents—one logical kind of fact, and whatever is a constituent of them.

My goal here is to show that the principle of atomicity is indefensible. Consideration of a certain kind of formula—generalized molecular ones—will show that we cannot get by with only atomic facts. I argue here that to give truth-makers for generalized molecular formulas, we need either general facts or molecular ones. A corollary of this is that if logical atomism is critically committed to the principle of atomicity, then logical atomism is not viable.[11]

2 The Argument for Dispensing with Molecular Facts

Russell's dispenses with molecular facts by assuming an ontology of atomic facts $R^n(a_1, ..., a_n)$ as truth-makers for atomic formulas.[12] This is unproblematic from the modern point of view in that it is standard among metaphysicists to posit something analogous to atomic facts.

With these raw materials, Russell argues that molecular facts are dispensable. He focuses on the case of molecular formulas whose constituent formulas are atomic.[13] He then argues that their truth-conditions are given entirely by the those of their constituent atomic formulas, plus the truth-tables for each connective.

[8] "...[the principle of atomicity] states that everything we wish to say can be said in sentences belonging to the 'atomistic hierarchy' which will be defined in section C of Chapter 13." (Russell, 1940/1973, 160)

[9] "...I shall call the assemblage of sentences obtained from atomic judgments of perception by the three operations of substitution, combination, and generalization, the *atomistic* hierarchy of sentences." (Russell, 1940/1973, 187)

[10] "Suppose that I am given *all* elementary propositions: then I can simply ask what propositions I can construct out of them. And there I have *all* propositions, and *that* fixes their limits. Propositions comprise all that follows from the totality of all elementary propositions (and, of course, from its being the *totality* of them *all*). (Thus, in a certain sense, it could be said that *all* propositions were generalizations of elementary propositions.)" (Wittgenstein, 1922/1971, 4.51-4.52)

[11] Note that logical atomism is arguably not committed to the principle of atomicity (Elkind, 2018, 29).

[12] "There you have a whole hierarchy of facts—facts in which you have a thing and a quality, two things and a relation, three things and a relation, four things and a relation, and so on. That whole hierarchy constitutes what I call *atomic* facts, and they are the simplest sort of fact...The propositions expressing them are what I call atomic propositions." (Russell, 1918/1986, 177)

[13] "But I am talking today about molecular propositions, and you will understand that you can make propositions with "or" and "and" and so forth, where the constituent propositions are not atomic, but for the moment we can confine ourselves to the case where the constituent propositions are atomic." (Russell, 1918/1986, 184-185)

For example, the disjunctive claim $p \vee q$ is true if there is an atomic fact corresponding to p or an atomic fact corresponding to q.[14] The truth-conditions for formulas involving the other binary truth-functional connectives are analogous to the case of '$\vee$' (Russell, 1918/1986, 185-186). In each case, we already have truth-makers for molecular formulas given an ontology of atomic facts and the truth-functional definition of each binary connective. This argument is seen as persuasive among truth-maker theorists today:

> Disjunctive sentences raise no special problems for the theory, since a disjunctive sentence is true only to the extent that one or other of its disjuncts is true... (Mulligan et al., 1984, 314)
>
> He [Russell] draws the line at disjunctive facts, for obvious reasons. All that is required for them is a truthmaker for at least one disjunct, and then there seems no need to postulate disjunctive facts in addition. (Armstrong, 2004, 54)
>
> There is consensus in the literature that not every proposition has its own disjunctive truthmaker. For instance, disjunctions are thought to be made true, *separately*, by the truthmakers for their disjuncts. Thus there is no need to postulate a distinctive kind of entity, like *disjunctive states of affairs*, that is supposed to make disjunctions true... (Rodriguez-Pereyra, 2006, 193)
>
> It's not unreasonable to think that no further truthmaker is needed for a conjunction than the truthmakers for each of its conjuncts, or that once you make a proposition true you thereby make true any disjunction of which that proposition is a disjunct... (Cameron, 2008, 411)
>
> ...once truth-makers have been supplied for the atomic truths, there is simply *no need* to posit further truth-makers for the molecular ones. All we need to recognise is that an atomic statement is true whenever a truth-maker for P exists, that P is false if and only if no truth-maker for P exists. Once the existence of and non-existence of the truth-makers has settled the truth-values of all atomic statements, the logical operations described by the truth-tables then settle the truth and falsity of all molecular statements... (MacBride, 2016, 36; see 25-26)

This consensus is, so far as the literature shows, widely-accepted. Now if we accept that, for any two facts, there is a fusion of them, then we have fusions of facts that look 'conjunctive' and 'disjunctive' (Jago, 2011, 44). But even fans of fused facts reject that there are facts having a conjunctive or disjunctive structure (Barker and Jago, 2012, 126).

At this juncture, the principle of atomicity is still viable: so far, only atomic facts are truth-makers. The above arguments, in my view, are persuasive, and welcome to those that wish to avoid molecular facts as I do. But note that they only deal with molecular formulas whose constituent formulas, ultimately, are atomic, such as '$Fa \vee Ga$'. For this reasoning to show that molecular facts are dispensable, we need to consider all molecular formulas, including those in which the constituent formulas are not atomic, such as '$\exists x[Fx] \vee Ga$'. We do this in §§4-5. In §3, we consider the argument for positing general facts

[14] "That is to say, the truth or falsehood of this proposition "p or q" depends upon two facts, and not upon one, as p does and as q does." (Russell, 1918/1986, 185)

3 The Argument for Positing General Facts

Russell next argues that general and existential formulas are made true by a special kind of fact, namely, *general and existence facts*, respectively. He first points out that we logically cannot infer any general formula merely on the basis of enumerating all its instances: for the general formula to follow, one needs the further claim that the enumerated instances are all of them.[15] So no general formula is logically implied by any collection of only atomic formulas. He then takes this claim as evidence that there are general facts. He writes:

> It is perfectly clear, I think, that when you have enumerated all the atomic facts in the world, it is a further fact about the world that those are all the atomic facts there are about the world, and that is just as much an objective fact about the world as any of them are. It is clear, I think, that you must admit general facts as distinct from and over and above particular facts. The same thing applies to "All men are mortal." When you have taken all the particular men that there are, and found each one of them severally to be mortal, it is definitely a new fact that all men are mortal; how new a fact, appears from what I said a moment ago, that it could not be inferred from the mortality of the several men that there are in the world. (Russell, 1918/1986, 207)

This argument may appear to rely on the following principle that many modern truth-maker theorists accept, *Truth-Maker Necessitarianism* (Rodriguez-Pereyra, 2006, 188):[16]

TMN Necessarily, if f is a truth-maker for ϕ, then that f exists implies ϕ.[17]

The word 'implies' here requires some qualification: Greg Restall in his critical discussion of **TMN** shows that the logical entailment cannot be right reading of 'implies' here (Restall, 2008, 89).[18] Those complications are discussed elsewhere (Rodriguez-Pereyra, 2006, §2). But we can set them aside here: whatever the right reading is, if the existence of a fact or a collection of them do not in some logical or metaphysical sense necessitate that a formula is true, then those facts are not truth-makers for the formula in question. The

[15] "You can never arrive at a general proposition by inference from particular propositions alone. You will always have to have at least one general proposition in your premisses. That illustrates, I think, various points. One, which is epistemological, is that if there is, as there seems to be, knowledge of general propositions, then there must be *primitive* knowledge of general propositions (I mean by that, knowledge of general propositions which is not obtained by inference), because if you can never infer a general proposition except from premisses of which one at least is general, it is clear that you can never have knowledge of such propositions by inference unless there is knowledge of some general propositions which is not by inference." (Russell, 1918/1986, 206)

[16] Russell himself would likely not endorse TMN because it is difficult to see how it could be explicated without involving some metaphysical necessity that is clearly not logical. Though the text is not explicit, Russell would likely have preferred an argument that invokes correspondence and uses a lack logical entailment from a formula ψ to some ϕ as indicating a lack of truth-making of ϕ by the facts corresponding to ψ. I thank Gregory Landini for bringing up this point in conversation.

[17] In symbols: $\Box\{(f \rightarrow_{\mathrm{TM}} \phi) \rightarrow_{\mathrm{L}} (\mathrm{E}!f \rightarrow_{\mathrm{L}} \phi)\}$. Here '$\rightarrow_{\mathrm{TM}}$' stands for the truth-making relation between a truth-maker and a truth-bearer, and '$\rightarrow_{\mathrm{L}}$' stands for logical entailment between truth-bearers. Note that we can eliminate the apparent term 'f' for a truth-maker, which f would presumably be logically complex, with $\exists R^n \exists a_1 ... \exists a_n [R^n(a_1, ..., a_n) \rightarrow_{\mathrm{TM}} \phi]$. Truth-maker Necessitarianism, in symbols, would then be: $\Box\{\exists R^n \exists a_1 ... \exists a_n [R^n(a_1, ..., a_n) \rightarrow_{\mathrm{TM}} \phi] \rightarrow_{\mathrm{L}} (\exists R^n \exists a_1 ... \exists a_n [R^n(a_1, ..., a_n)] \rightarrow_{\mathrm{L}} \phi)\}$.

[18] Lewis for his part proposes **TMD**: for any two worlds w and v, if ϕ is true in w but not true in v, then some f exists in w but not in v (Lewis, 2001, 606). See also (Armstrong, 2004, 69).

ontological role of a truth-maker is to suffice for the truth of some corresponding formula. If the given fact does not do even this, it is not worthy of the truth-maker name.

Such is Russell's point in discussing a collection of particular facts, like *Russell being human* and *Russell being mortal*, *Wittgenstein being human* and *Wittgenstein being mortal*, and so on. These facts alone do not necessitate that *all humans are mortal*. So they are not truth-makers for the claim that *all humans are mortal*. Just atomic facts are not enough. One further needs a general claim, and its corresponding fact, to necessitate any general claim. So we have to posit at least one truth-maker of a general character.

Utilizing TMN, the Russellian argument can be summarized as follows:

The Russellian Argument for General Facts

(1)	**TMN** holds and no class of atomic facts necessitates $\forall x[Hx \rightarrow Mx]$.	Premise
(2)	If **TMN** holds and no class of atomic facts necessitates $\forall x[Hx \rightarrow Mx]$, then there are non-atomic facts as truth-makers.	Premise
(3)	So, there are non-atomic facts as truth-makers.	By 1-2

I expect that many other truth-maker theorists find this argument compelling. Note that if this argument is right, the principle of atomicity has to be abandoned: we have non-atomic facts as truth-makers. Such facts presumably have a general character or logical form,where 'general fact' here is shorthand for either a universal or existential fact.

But the stranger point is yet to come. As I argue below, given certain assumptions about whether an empty world is possible, a truth-maker theorist is committed to either general facts or molecular facts—either the one or the other. That is, the issues of whether there are general facts or molecular facts are quite closely interrelated, as we will see.

4 Incompletely Generalized Molecular Formulas

Russell was among the first to give the now widely-accepted argument dispensing with molecular facts. He also noticed that the argument is difficult to extend to cases where the constituents of molecular formulas are not atomic. Russell wrote in Lecture V:

> There is one point about whether there are molecular facts. I think I mentioned, when I was saying that I did not think there were disjunctive facts, that a certain difficulty does arise in regard to general facts. Take "All men are mortal." That means: 'x is a man' implies 'x is a mortal' whatever x may be."...It is perhaps a little difficult to see how that can be true if one is going to say that "'Socrates is a man' implies 'Socrates is mortal'" is not itself a fact, which is what I suggested when I was discussing disjunctive facts. I do not feel sure that you could not get round that difficulty. I only suggest it as a point which should be considered when one is

> denying that there are molecular facts, since, if it cannot be got round, we shall have to admit molecular facts. (Russell, 1918/1986, 208)

Russell's concern is that the truth-conditions of a formula such as 'for all x, if x is human, then x is mortal' cannot be explained merely through atomic facts. This is because, unlike in the case of 'Hs implies Ms'—Socrates is human implies Socrates is mortal—the molecular formula occurs within the scope of a formula, and its terms are bound by an initial quantifier. That is, while the truth-conditions of 'Hs implies Ms' are given by those of its atomic constituents, the formula '$\forall x[Hx \rightarrow Mx]$' is not: it is not equivalent either to '$\forall x[Hx]$ implies $\forall x[Mx]$' or to any collection of instances of 'Ha implies Ma', 'Hb implies Mb', and so on. So one might be tempted to think that the 'molecularity' of the formula '$\forall x[Hx \rightarrow Mx]$' is in a sense trapped by the universal quantifier, so that it seems to have a molecular truth-maker.

One can see how Russell in 1918 was concerned about this. But today we are blessed with model-theoretic semantics: in 1933, Tarski showed how to recursively define the truth of quantified formulas through their instances. One simply says that a universally quantified formula $\phi(x)$ is satisfied if and only if every assignment of the variable x to some object (in the domain) results in a true formula (Mendelson, 1987, 47-48). And so supposing that the truth-conditions of the formula '$\forall x[Hx \rightarrow Mx]$' are completely given by model-theoretic account of its truth as satisfaction, the generalized molecular formula's truth is given by the truth-conditions of all its instances, which in turn are given by the truth-table for implication. So we once again have all and only atomic formulas as truth-makers for molecular formulas.

But this reasoning, as we will see in the next section, only plausibly covers one of two cases. To distinguish these two cases, we need some terminology. A *generalized* molecular formula is one in which a molecular formula '$\phi(x) \rightarrow \psi(x)$' occurs within the scope of a quantifier '$\forall$' and '$\forall$' binds some variable term 'x' occurring in '$\phi(x)$' and in '$\psi(x)$'. So a generalized molecular formula has the form $\forall x[\phi(x) \rightarrow \psi(x)]$.[19] This terminology is meant to capture the kinds of formulas that worried Russell: generalized molecular formulas have the feature that they appear to be ineliminably molecular because the quantifier traps the truth-functional connective in smallest scope. For a generalized molecular formula lacks, or appears to lack, an equivalent formula wherein '$\rightarrow$' has the primary or widest scope.

Now for a formula to be ineliminably molecular, the quantifier has to bind a variable occurring in both parts of the molecular formula. An example is '$\forall x[Hx \rightarrow Mx]$'. If instead a quantifier only bound a variable term in one half of the molecular formula, as in '$\forall x[Hs \rightarrow Mx]$', then we get a formula equivalent to '$Hs \rightarrow \forall x[Mx]$', which is not the sort of formula that worried Russell. He was concerned with formulas that

[19] A similar definition can be given for the other binary truth-functional connectives. We list them for the most common connectives here: $\forall x[\phi(x) \vee \psi(x)]$, $\exists x[\phi(x) \wedge \psi(x)]$, and $\forall x[\phi(x) \leftrightarrow \psi(x)]$.

seem *ineliminably* molecular like '$\forall x[Hx \rightarrow Mx]$'. On the other hand, '$\exists x[Hx \rightarrow Mx]$' is not ineliminably molecular because this is equivalent to '$\exists x[\neg Hx] \vee \exists x[Mx]$'. But the quantifier Q may be existential or universal, and the connective may instead be '$\vee$' or '$\leftrightarrow$', with the caveat that the formula must be ineliminably molecular: again, it is not equivalent to a formula in which the truth-functional connective has primary scope over the quantifier's scope.

Having characterized generalized molecular formulas, we next distinguish their two kinds. An *incompletely generalized* molecular formula is a generalized molecular formula in which some non-variable term occurs. Non-variable terms include the singular term 'Socrates' or 's', the predicate 'human' or 'H', the predicate 'mortal' or 'M', and so on. This is the case that Russell considers in his 1918 discussion, and it is the case that Tarski solved in the 1930s. The model-theoretic account of truth as satisfaction supplies truth-makers for incompletely generalized molecular formulas: these will be the truth-makers for each molecular instance of the formula, which are all and only atomic facts, plus the non-atomic fact—a general fact—that these are all the instances of the generalized formula.

But invoking model-theoretic semantics only partly accounts for the truth of generalized molecular formulas: there remains another, more problematic kind of generalized formula.

5 Completely Generalized Molecular Formulas

A *completely generalized* molecular formula is a generalized molecular formula in which only variable terms occur, like '$\forall F \exists G \forall x[Fx \rightarrow Gx]$' or '$\forall F \forall x[Fx \rightarrow Fx]$'.[20] In this section I argue that if we want to avoid molecular facts, we will need non-atomic truth-makers for such formulas: considerations of such formulas will show that we need a new kind of fact—"a new beast for our Zoo" (Russell, 1918/1986, 199)—as a truth-maker for such formulas.

A truth-maker corresponding to this ineliminably molecular formula has an unattractive molecularity. To avoid such truth-makers, in the next section I show how to define away this kind of formula, eliminating the apparently ineliminable molecularity of completely generalized molecular formulas, so that there are no completely generalized molecular formulas at all. They will be defined in terms of formulas whose molecularity is eliminable by truth-functional considerations—the very considerations that truth-maker theorists from Russell to the present day invoke in the case of formulas like '$Hs \rightarrow Ms$'.[21]

[20] "Now I want to come to the subject of *completely general* propositions and propositional functions. By those I mean propositions and propositional functions that contain only variables and nothing else at all. This covers the whole of logic." (Russell, 1918/1986, 208)

[21] The cost of this way out is that we must embrace general facts as truth-makers for completely generalized atomic formulas,

The problem raised by completely generalized molecular formulas is that, in some cases, their truth is necessitated without any instances. Russell implicitly raises this kind of case:

> I want to say emphatically that general propositions are to be interpreted as not involving existence. When I say, for instance, "All Greeks are men", I do not want you to suppose that this implies that there are Greeks. It is to be considered emphatically as not implying that...If it happened that there were no Greeks, both the proposition that "All Greeks are men" and the proposition that "No Greeks are men" would be true. (Russell, 1918/1986, 201-202)

This goes hand-in-hand with Russell's immediately preceding remark, "All general propositions deny the existence of something or other. If you say "All men are mortal", that denies the existence of an immortal man, and so on." (Russell, 1918/1986, 201) Russell is raising the possibility of general claims being true despite having no instances.

This point is critical because truth-maker theorists have seemingly taken the view that one general fact was sufficient to explain generalized molecular truths: this is "the general fact that all the facts (states of affairs) of lower order *are* all such facts." (Armstrong, 2004, 74) This account seems plausible for incompletely generalized molecular formulas: it seems at least defensible that 'all humans are mortals', like 'no humans are mortals', is vacuously true in a world without humans: in a world without any humans, there are no instances, so both contrary claims are made true by the mereological sum that is the second-order general fact. This suggestion is attractive: we only get general facts as parasitic on atomic facts.

But this suggestion is untenable when the world is empty of concrete entities, so that there are no atomic facts—and thus nothing for Armstrongian general facts to be parasitic upon.[22] In an empty world, there are neither atomic facts nor mereological sums of them, and so we have no truth-makers for true completely generalized molecular formulas save through positing generalized molecular facts as *sui generis* entities.

So we seem stuck again with ineliminably molecular facts as truth-makers for completely generalized molecular formulas. The argument for this is as follows:[23]

An Argument for Molecular Facts

(1)	An empty world is logically or metaphysically possible.	Premise
(2)	Some completely generalized molecular formulas are true in an empty world ($\forall F \exists G \forall x[Fx \rightarrow Gx]$) and others are not ($\exists F \forall G \forall x[Fx \rightarrow Gx]$).	Premise
(3)	There are neither atomic facts nor sums of them in an empty world.	Premise
(4)	True completely generalized molecular formulas need truth-makers.	Premise

that is, we must embrace general facts of an apparently atomic character that lack any constituents at all, which I call *completely general facts* or *facts with no constituents*. I discuss this in §8.

[22]Even in an empty world, however, there may be atomic facts involving *ante rem* entities like logical universals. I address this reply below.

[23]I thank Peter Simons for some helpful comments on an earlier version of my argument.

(5) If (1), (2), (3), and (4), then true completely generalized molecular formulas have non-atomic truth-makers of a molecular character. Premise

(6) So, true completely generalized molecular formulas have non-atomic truth-makers of a molecular character. By 1-5

Such is the argument that, I think, is a sound argument for completely general facts. Because molecular truth-makers cause such widespread revulsion, it is worth considering the grounds for accepting these premises. In the next section, I defend the argument's premises.

6 Metaphysical Nihilism and Empty Domains

Premise (5) is manifestly true once (1)-through-(4) are accepted. So one might first try to deny premise (4): completely generalized molecular formulas do not need truth-makers at all. It might be suggested that such formulas are vacuously true.

But such formulas cannot be vacuous. For some of them are theorems of logic, while others are disprovable by logic alone. There is no characterization of these two classes of formulas, consistent with their supposed vacuity, that accounts for why any formula in one class is true while any in the other is false.

For example, suppose that we follow Quine's way of dealing with the empty domain. Let all universally bound formulas be true, and all existentially bound formulas be false (Quine, 1954, 177). Now the formulas '$\forall F \exists G \forall x[Fx \rightarrow Gx]$' or '$\forall F \forall x[Fx \rightarrow Fx]$' are both theorems of standard second-order quantification theory. Further, on Quine's approach, they are also theorems in empty domain logics, or *inclusive logics*: they are provable even in a logic consistent with an empty world.

Contrast these formulas with those like '$\exists F \exists G \forall x[Fx \rightarrow Gx]$' and '$\exists F \forall x[Fx \vee \neg Fx]$'. These formulas are just closures of tautologies like '$Pa \rightarrow Pa$' and '$Pa \vee \neg Pa$'. Somehow, these formulas are to be false—but vacuously so—while universal closures of tautologies are true—but also vacuously so. But if, by their vacuity, no truth-maker is needed for the first class of formulas, then why should the other kind not be, by their vacuity, true?

Quine's answer to this question is that vacuous formulas are justified by their logical equivalence to vacuity-free formulas (Quine, 1954, 178). For example, $\exists x(\phi)$ is logically equivalent to $\exists x(\phi \wedge Fx \rightarrow Fx)$ (where ϕ is a formula in which x does not occur). Thus the vacuous falsehood of $\exists x(\phi)$ in the empty domain is accounted for by its equivalence to the non-vacuous falsehood of $\exists x(\phi \wedge Fx \rightarrow Fx)$.[24]

But this answer just pushes the problem back. It is widely agreed that every true formula needs a truth-

[24]The semantics and proof theory for empty domain logics still has no standard semantics and proof theory, though typically Quine's treatment is followed (Williamson, 1999, 3-4).

maker. So some truth-maker is needed in the empty domain to account for either the vacuous formula's truth or that of its non-vacuous equivalent. Quine's clever proposal permits us to distinguish vacuously true formulas from vacuously false ones in a principled way. But distinguishing true and false vacuous formulas using their non-vacuous equivalents does not address the underlying problem: indeed, Quine's proposal relies on these vacuity-free formulas being non-vacuously true. So appealing to vacuity to resist premise (4) will not work: completely generalized molecular formulas, if true, are true non-vacuously.[25] As such, a truth-maker theorist needs truth-makers for them. This is just premise (4).

One might object to premise (3), that there are no atomic facts in an empty world, by arguing that this only holds of atomic facts involving concrete entities.[26] But if one embraces universals existing outside space-time, and holds that these exist necessarily, then there could be atomic facts involving relations between properties and properties of properties. That is, if one embraces *ante rem* realism about universals, then it could be that there are atomic facts having the form $R^n(F_1, ..., F_n)$ even in an empty world.

I am comfortable conceding that *ante rem* realism about universals is a way out of my argument for facts corresponding to generalized molecular formulas. If one holds that universals necessarily exist even when they have no instances, then one has grounds for rejecting premise (3).[27] But it is still interesting to establish that it is either *ante rem* realism about universals or non-atomic facts. This is because there are serious worries about positing *ante rem* universals. Such entities belong to a distinct ontological category from particulars, so positing them seems transgresses such parsimony considerations as motivate nominalism and are *prima facie* principles for good metaphysical practice.[28] Additionally, *ante rem* universals are non-causal entities existing outside space-time, so positing them seems to run afoul of such methodological principles as underlie metaphysical naturalism.[29] Further, as we do not causally interact with *ante rem* universals, our knowledge of them is *prima facie* difficult to explain.[30] So premise (3) holds, unless one rejects it by

[25]There is a further argument against such formulas being true or false vacuously. There are different and sometimes contradictory logics. We do not wish to say that these logics are all equally true or false. But this seems to be what we are forced to say because they are equally vacuous in the empty world. This has the unhappy corollary that such debates are trivialized into disputes over equally vacuous systems. The view that logicians are rather disagreeing over the facts—over what facts obtain of logical necessity not just when there are concrete things, but in every case—is thus more consonant with logicians' internal point of view.

[26]I thank Mark Textor for pointing this out to me in conversation.

[27]We will see in §8 that supplying truth-makers for completely generalized molecular formulas requires accepting abstract entities a different, more plausibly logical sort—completely general facts, which have no constituents. These *sui generis* entities will exist necessarily even when there are no atomic facts falling under them and no concrete objects, as in the empty world. This seems more palatable to me than accepting an ontology of non-logical but necessarily existing universals with no instances. The exact list of them depends on one's other philosophical commitments. But the candidates like *being* H_2O, *being human*, *being siblings*, *being taller than*, *having negative charge*, and so on, all seem to be problematically non-logical entities, as do the atomic facts involving them and involved as truth-makers for general formulas. Facts with no constituents have their own difficulties, but they are not problematically non-logical.

[28]See, for example, (Williams, 1997, 118-120), but see also (Campbell, 1997, 133-134).

[29]See, for example, (Armstrong, 1978a, 22-24, 75) and (Armstrong, 1978b, 8-11).

[30]See, for example, (Benacerraf, 1973, 671-673) and (Shoemaker, 1997, 236-237).

embracing *ante rem* universals, and this alternative has its own serious difficulties.[31]

As for premise (2), the only way to deny this is by asserting that an empty domain logic is *trivial*. There are two senses in which a system might be trivial. In the usual sense, a trivial system has that every well-formed formula is a theorem. Now this is provably false. There are multiple non-trivial, sound, and complete proof systems for inclusive logic.[32] Though there are still outstanding issues in empty domain logic, we know at least that it is non-trivial inasmuch as not every formula is a theorem in the empty world.

Alternatively, a system might be trivial in the sense that no well-formed formulas are theorems. The motivation for this would be that, in an empty world, every well-formed formula of the system is false. As such, no generalized molecular formulas should be derivable. A truth-maker theorist might find this suggestion attractive because, after all, there are no truth-makers in empty worlds, so it might seem sensible that all formulas should be false.

The plausibility of this reply depends on denying premise (1). If the empty world is logically possible, then holding that all formulas are false in the empty world makes all logical laws false in the empty world.[33] Take the law of tautology for example: on this reply, for any well-formed formula ϕ, $(\phi \wedge \phi) \rightarrow \phi$ is now false in the empty world. And yet the law of tautology is true in all non-empty worlds. This point applies equally to any law of logic. So assuming premise (1) is independently plausible, the reply makes all laws of logic logically contingent. But this is contrary to the view that there is at least one law of logic that is logically necessary.[34] So if premise (1) is independently defensible, then, assuming that there at least one law of logic is logically necessary, which I think readers will agree to, we have premise (2).[35]

Premise (1) is the claim that an empty world is either logically or metaphysically possible. Premise (1) is likely the most controversial premise in the argument. Part of the reason for this is that philosophers disagree over whether logical or metaphysical possibility is the widest, all-encompassing, absolute sense of 'possible' is.[36] Without taking a stance on this debate, I want to argue that the empty world is possible in the widest sense of 'possible' whatever that is—hence the inclusive 'or' in premise (1). So I will first argue

[31] See, for instance, (Armstrong, 1997, §2.8).

[32] Confer (Mendelson, 1997, §2.16) and (Williamson, 1999, 4).

[33] By a *law of logic*, here I mean an axiom schemata of a logical system and not a principle of inference. The objection in this paragraph is not about principles of inference: for any principle of inference would vacuously take all truths to truths in the empty world, since the reply assumes that all formulas are false.

[34] One might deny that there are any axiom schemata that are logically necessary (Maddy, 2014, 99-100). This is another way out of my argument, but deflating logical and metaphysical necessity is not an attractive strategy for a truth-maker theorist.

[35] There is yet another way of attacking premise (2). One could argue that all formulas in an empty world would be neither true nor false. They might be without truth-values at all or else they might have some truth-value other than true and false. Some free logics allow for developments along these lines (Lambert, 2003, 127). However, the argument in this paragraph applies to such systems: the laws of logic would turn out neither true nor false in the empty world, though they would be true in other worlds. Deploying supervaluations will not undercut the argument of this paragraph: even with supervaluations, either all completely generalized molecular formulas will be true or all of them will be false, on pain of admitting premise (2) is true. This seems to make the laws of logic logically contingent.

[36] See (Nolan, 2011, 315-323) for a nice discussion of this issue.

that the empty world is metaphysically possible, and then that it is logically possibility.

I begin with the metaphysical possibility of the empty world.[37] It is usual today to understand talk of metaphysical possibility through accounts of how we should understand possible worlds, especially whether talk of worlds is understood in terms of modal operators or, conversely, whether modal operators are understood through talk of possible worlds (Williamson, 2013, 333). So the defense of premise (1) in the metaphysical sense of 'possible' can be made by showing that it is possible on various accounts of possible worlds. And the case for premise (1) is strengthened by noting that the standard *ersatz* accounts of metaphysical possibility are such that premise (1) holds.

Consider some accounts of possible worlds: possible worlds might be concrete situations (as Lewis has it), or abstract entities like maximal collections of propositions (as Fine has it), or stipulated situations (as Kripke has it), or combinations of metaphysically simple entities (as Armstrong has it), or as fictions (as Rosen has it).[38] These accounts might be broadly categorized as *realist* or *ersatz* according to whether possible worlds are genuine entities or not (Parent, 2012). On all of the usual *ersatz* accounts, including fictionalist ones for the moment, it is metaphysically possible that there should be empty worlds (Coggins, 2010, 138).

For example, to get a maximal set of propositions that would be entailed in the empty world, we can follow Quine in taking the universal closure of all formulas. One may wish to secure a 'more' maximal set of propositions by adding, in the spirit of free logic, the negations of all atomic formulas like $R(a_1, ..., a_n)$ even though $a_1, ..., a_n$ do not refer. This can easily be done, giving us a maximal set of propositions. As such, on the *ersatz* view that a possible world is a maximal set of propositions, there is an empty world. As another example, the modal fictionalist would doubtless concede the fictional character of the empty world. Even the Kripkean account of stipulated worlds seems to permit the stipulation of *a world just like this one, except containing no entity at all.*[39]

In contrast, those with realist accounts of metaphysical possibility in terms of really existing, concrete, possible worlds are in a position to reject premise (1) (Lewis, 1968, 73-74). I concede this point. But absent an embrace of a realist account of possible worlds, which has its own problems, the usual views of metaphysical possibility using *ersatz* accounts of possible worlds are such that premise (1) in the sense of

[37] There has already been substantial debate over premise (1): in the sense of metaphysical possibility, premise (1) is also known as *metaphysical nihilism*. Much discussion has concerned the so-called *subtraction argument* for metaphysical nihilism (Efird and Stoneham, 2009, 132-133). I will not add to the subtraction argument here. For detailed discussion, see (Coggins, 2010, Chapter 6).

[38] A discussion of various accounts is found in (Menzel, 2017, §2).

[39] "A possible world is *given by the descriptive conditions we associate with it*...'Possible worlds' are *stipulated*, not *discovered* by powerful telescopes." (Kripke, 1980, 44)

metaphysical possibility has to be conceded.

Next, consider premise (1) in the sense of logical possibility. Here we must distinguish logical possibility from the existence of a model: there are models of $Fa \wedge \neg Fa$, but this fact alone does not show that contradictions are logically possible.[40] The guarantee of a model is relative to some logical system. We are concerned with logical possibility in a broadest, non-relative sense.[41]

So taking 'possible' in the broadest, logical sense, there are two reasons to think that empty worlds are possible. First, it is widely though not universally held that logic does not show, and should not have as a theorem, that anything exists.[42] This is why logicians since Russell have hesitated over existence theorems in logic generally, and particularly in regards to what actual concreta exist in the universe.[43] This view of logic as independent of existential claims strongly supports premise (1).

Second, premise (1) is supported the notion of logical form. Typically, logic is described as being concerned with the form or structure of an argument rather than with its content or its premises' truth.[44] But logical form alone does not show that empty worlds are impossible: purely formal matters are independent of how the world is.[45] To take a well-known example, the inference from $\forall x[\phi(x) \rightarrow \phi(x)]$ to $\exists x[\phi(x) \rightarrow \phi(x)]$ is valid in classical logic, but this is only valid if the domain is assumed to be non-empty. This assumption is standard, but it is generally maintained that this assumption is not imposed by the argument's form. It is not held that there is a formal feature of the thesis that $\forall x[\phi(x) \rightarrow \phi(x)]$ such that it follows, as a matter of logical form, that $\exists x[\phi(x) \rightarrow \phi(x)]$. Rather, non-empty domains are usually justified, and have been since Quine, as a matter of technical convenience.[46] However, on the usual understanding of logical form, there is no reason to believe that $\forall x[\phi(x) \rightarrow \phi(x)]$ requires for its truth that there be some a that satisfies

[40] I thank a referee for pressing me to clarify this point.

[41] It is also established that inclusive logic has a model and is sound and complete with respect to some of its proof systems and formal semantics (Mendelson, 1997, §2.16). So if 'logical possibility' just meant something system-relative, like having a model or entailing no contradiction, then premise (1) follows anyway.

[42] If one embraces *necessitism*—the view that, necessarily, any x is such that, necessarily, some y is identical to x—then premise (1) can and must be rejected. But the above argument takes truth-maker theory as a premise, and necessitism is inconsistent with truth-maker theory (Williamson, 2013, 391-392). So for truth-maker theorists who already must reject necessitism, this way of resisting premise (1) is closed.

[43] See (Russell, 1937, §5) and (Russell, 1919, 203).

[44] I cite five entirely typical examples. "The argument, it may be held, is valid from its form alone, independently of the matter, and independently in particular of the question whether the premisses and the conclusion are in themselves right or wrong." (Church, 1956, 2) "The truth or falsity of the particular premisses and conclusions is of no concern to logicians. They want to know only whether the premisses imply the conclusion." (Mendelson, 1997, 1) "To sum up, formal logic is fundamentally concerned with the form and structure of arguments and not, primarily, with their content." (Tomassi, 1999, 17) "[A] formal derivation exploits only the shape of formulas, not any consideration of their truth or falsity." (Goldrei, 2005, 87) "It is important to remember that when we evaluate arguments, we must always distinguish truth value analysis from the logical analysis." (Baronett, 2013, 30)

[45] "Thus the absence of all mention of particular things or properties in logic or pure mathematics is a necessary result of the fact that this study is, as we say, "purely formal."" (Russell, 1919, 198)

[46] Quine, for example, argues against including the empty domain because doing so "would mean surrendering some formulas which are valid everywhere else and thus generally useful." (Quine, 1954, 177) See also (Hunter, 1973, 255), (Mendelson, 1997, 147), and (Tomassi, 1999, 291-292). It should be noted that there are some "subtleties" involved in the semantics for empty domains (Williamson, 1999, 3).

$\phi(a) \rightarrow \phi(a)$.[47] So for these two reasons, we must admit that empty worlds are logically possible. This gives us premise (1) in the sense of logical possibility, which was our final premise. So the argument is sound.

To sum up: the argument is decisive for any truth-maker theorists who (a) admit the logical or metaphysical possibility of an empty world, (b) accept that not every completely generalized molecular formula is a theorem of inclusive logic, and (c) deny *ante rem* realism about non-logical universals. Any such truth-maker theorists are thereby committed to positing non-atomic facts as truth-makers for completely generalized molecular formulas.

7 Back to 1918: Completely General Facts

We saw in the previous section that there are non-atomic facts as truth-makers for completely generalized molecular formulas. Such formulas seem more intractable for a truth-maker theorist than has been generally acknowledged: as we saw, they are not reducible to the case of incompletely generalized molecular formulas if empty worlds are possible. Some facts must be supplied for such formulas.

What sort of facts are these? We noted that there are no atomic facts in an empty world. So the truth-makers for true completely generalized molecular formulas apparently are non-atomic. Indeed, they seem to be molecular facts. *Pace* (Barker and Jago, 2012), positing such facts is an unappealing move. But can we avoid positing molecular facts?

The natural solution to this problem is to get rid of completely generalized molecular formulas. One would define formulas like '$\forall F \exists G \forall x[Fx \rightarrow Gx]$' in such a way that no molecular expressions occur within the scope of a quantifier. Then truth-functional considerations would apply so no molecular facts would be needed to account the truth of completely generalized molecular formulas. This would be a symbolic dissolution to the problem of providing truth-makers for completely generalized molecular formulas.[48]

Such a symbolic dissolution is not possible. Quantifiers at the front of a formula cannot in general be brought into the scope of molecular connectives occurring in their scope.[49] For a counterexample, there is no formula equivalent to $\forall F \forall x[Fx \vee \neg Fx]$ which is such that no molecular connective occurs in the scope

[47] Indeed, to say otherwise jars with the standard history that logic since Boole has made an advance in rejecting the traditional view that "Some s are P" follows from "All s are P."

[48] There are some quantifier-like notions that are not covered by the proposals given below. For example, the natural language notion of *most* seemingly is not translatable into first-order language using only '$\forall$', '$\exists$', and the usual truth-functional connectives (Stevens, 2011, 112-113). This raises a question: could one such as Russell, with the higher-order language of *Principia* and class talk in hand, express the truth conditions of natural language sentences involving quantifier-like notions, such as *most philosophers are wise*? I do not address that question here, but if the answer is 'yes', then what I say here will extend to quantifier-like notions like *most*. If the answer is 'no', then additional steps will be needed for other quantifier-like notions.

[49] This is possible in some cases, as in $\forall F \forall G \forall x[Fx \wedge Gx]$ because $\forall$ distributes over $\wedge$. Since this formula is logically equivalent to $\forall F \forall x[Fx] \wedge \forall G \forall x[Gx]$, we can define the former using the latter.

of a quantifier by the usual quantifier distribution laws.[50] Since we cannot get rid of the molecularity in completely generalized molecular formulas by defining such formulas away, the truth-maker theorist must address them: some non-atomic truth-makers must be supplied in light of the argument in §6.

One way of addressing the problem would be to supply truth-makers for completely generalized molecular formulas that are non-molecular. The natural candidate truth-makers are general facts. But these will be an unusual sort of fact. They will need to be facts that make true formulas with only variable terms and logical constants. Since some such formulas will be true in an empty world, these general facts will need to exist in an empty world. As such, these truth-makers will seemingly be facts—complexes—with no constituents.

Furthermore, as Lewis notes, there are no mereological sums of anything in empty worlds. So such general facts will not be mereological sums as Armstrong has it. Such general facts are *sui generis* entities and are not amenable to definition. They are the non-molecular facts that are, by the argument above, needed as truth-makers for the *definiens* of completely generalized formulas.[51] Call these *completely general facts*.[52]

Crucially, completely general facts do not have a structure: indeed, they have no constituents or parts in any sense, even though the formulas that pick them out have linguistic components. So they can serve as truth-makers for completely generalized molecular formulas without leading one to embrace molecular facts. In this way, completely general facts allow one to solve the problem of providing non-molecular truth-makers for completely generalized molecular formulas: although such facts are non-atomic, they are not molecular because they do not have a structure at all: rather, such facts *are* logical structures.[53]

Now Russell already commits to completely general facts in the 1918 lectures on logical atomism: such facts are picked out with formulas containing only variable terms and logical constants according to Russell.[54] Their complete generality—their avoidance of mentioning any particular things in the world—gives general facts a distinctly logical character that Russell exploits in accounting for the truth of logical principles. Russell says, "All the statements of logic are of that sort." (Russell, 1918/1986, 209) So Russell has the

[50] It is true that any formula in a classical logic is equivalent to some formula with all its quantifiers occurring at the front of the formula; such a formula is said to be in *prenex normal form* (Mendelson, 1987, §2.10).

[51] An alternative characterization would be that they are facts such that they make true some formula with only variable terms.

[52] Putting all formulas into their prenex normal form, completely general facts will be of different sorts according to the initial binding quantifier. For example, *completely general universal facts* are the would-be truth-makers for formulas with an initial universal quantifier. *Completely general existence facts* are the would-be truth-makers for formulas prefixed by an existential quantifier.

[53] See (Russell, 1913/1983, 114). But the comparison between Russell's notion of logical form and his posit of completely general facts should not be overemphasized: his discussion of logical form is brief and highly tentative (Griffin, 1980, 117). See (Griffin, 1980, 144, 152) and (Klement, 2015, 216).

[54] "Now I want to come to the subject of *completely general* propositions and propositional functions. By those I mean propositions and propositional functions that contain only variables and nothing else at all. This covers the whole of logic. Every logical proposition consists wholly and solely of variables, though it is not true that every proposition consisting wholly and solely of variables is logical." (Russell, 1918/1986, 208)

materials required to solve the problem of completely generalized molecular formulas.[55]

So if one has independent reasons to posit completely general facts, like the argument of §5, then it makes sense to put these posits to work. Completely general facts, besides being the facts of logic itself, will be truth-makers for completely generalized molecular formulas. Further, the discovery and categorization of such facts is an explicit aim of logical atomism:

> In logic you are concerned with the forms of facts, with getting hold of the different sorts of facts, different *logical* sorts of facts, that there are in the world. (Russell, 1918/1986, 191)

In contrast, the principle of atomicity is implausible absent *ante rem* universals, realism about possible worlds, or trivialization of inclusive logic, given the severity of Russell's problem of completely generalized molecular formulas. Furthermore, the principle of atomicity is not crucial to logical atomism: it is rather one among many views as to the results of inquiry into what logical kinds of facts there are, and it is a view that Russell himself did not hold in 1918. Now if non-atomic sorts of facts must be admitted in light of Russell's problem of generalized molecular formulas, then with completely general facts in our ontology, we avoid molecular facts while solving the problem of completely general formulas. More ambitiously, logic may be naturally identified, as Russell explicitly does, with the study of such facts. Enter logical atomism.[56]

References

D. M. Armstrong. *Nominalism & Realism: Universals & Scientific Realism*, volume 1. Cambridge University Press, New York, 1978a.

D. M. Armstrong. *A Theory of Universals: Universals & Scientific Realism*, volume 2. Cambridge University Press, New York, 1978b.

D. M. Armstrong. *A World of States of Affairs*. Cambridge University Press, Cambridge, 1997.

D. M. Armstrong. *Truth and Truthmakers*. Cambridge Studies in Philosophy. Cambridge University Press, Cambridge, 2004.

Stephen Barker and Mark Jago. Being Positive About Negative Facts. *Philosophy and Phenomenological Research*, LXXXV(1):117–138, July 2012. doi: 10.1111/j.1933-1592.2010.00479.x.

Stan Baronett. *Logic*. Oxford University Press, second edition, 2013.

Paul Benacerraf. Mathematical Truth. *The Journal of Philosophy*, 70(19):661–679, 1973. URL `https://www.jstor.org/stable/2025075`.

Ross P. Cameron. How to be a Truthmaker Maximalist. *Noûs*, 42(3):410–421, September 2008.

[55] It is reasonable to ask why Russell did not solve this problem if he had all the materials for the solution available to him. But Russell did not cleanly separate the two problems as I have done here, and completely general facts would not assist in solving the problem of incompletely generalized molecular formulas—and this is the only side of the problem that Russell explicitly discusses in 1918.

[56] ACKNOWLEDGMENTS.

Keith Campbell. The Metaphysic of Abstract Particulars. In D. H. Mellor and Alex Oliver, editors, *Properties*, chapter X, pages 125–139. Oxford University Press, 1997. Reprinted from Peter French *et al.* (1981) *Midwest Studied in Philosophy VI*.

Alonzo Church. *Introduction to Mathematical Logic*, volume 1 of *Princeton Mathematical Series*. Princeton University Press, 1956.

Nino B. Cocchiarella. *Formal Ontology and Conceptual Realism*, volume 339 of *Studies in Epistemology, Logic, Methodology, and Philosophy of Science*. Springer, 2007.

Geraldine Coggins. *Could There Have Been Nothing? Against Metaphysical Nihilism*. Palgrave Macmillan, 2010.

Marian David. The Correspondence Theory of Truth. *Stanford Encyclopedia of Philosophy*, Fall 2016. URL `http://plato.stanford.edu/archives/fall2016/entries/truth-correspondence/`.

David Efird and Tom Stoneham. Justifying Metaphysical Nihilism: A Response to Cameron. *The Philosophical Quarterly*, 59:132–137, 2009. URL `https://www.jstor.org/stable/40208582`.

Landon D. C. Elkind. On Russell's Logical Atomism. In Landon D. C. Elkind and Gregory Landini, editors, *The Philosophy of Logical Atomism: A Centenary Reappraisal*, History of Analytical Philosophy, chapter 1, pages 3–37. Palgrave MacMillan, London, 2018.

Derek Goldrei. *Propositional and Predicate Calculus*. CRC Press, 2005.

Nicholas Griffin. Russell on the Nature of Logic. *Synthese*, 45(1):117–188, September 1980.

Geoffrey Hunter. *Metalogic: An Introduction to the Metatheory of Standard First Order Logic*. University of California Press, 1973.

Mark Jago. Setting the Facts Straight. *Journal of Philosophical Logic*, 40(1):33–54, 2011. URL `http://www.jstor.org/stable/41487502`.

Kevin C. Klement. The Constituents of the Propositions of Logic. In Donovan Wishon and Bernard Linsky, editors, *Acquaintance, Knowledge, and Logic: New Essays on Bertrand Russell's* The Problems of Philosophy, chapter 10, pages 189–230. CSLI Publications, 1 edition, 2015.

Saul Kripke. *Naming and Necessity*. Harvard University Press, 1980.

Karel Lambert. The Philosophical Foundations of Free Logic. In *Free Logic: Selected Essays*, chapter 8, pages 122–175. Cambridge University Press, 2003.

David Lewis. Truthmaking and Difference-Making. *Noûs*, 35(4):602–615, 2001. URL `https://www.jstor.org/stable/2671864`.

David K. Lewis. Counterpart Theory and Quantified Modal Logic. *The Journal of Philosophy*, 65:113–126, 1968.

Fraser MacBride. Truthmakers. *Stanford Encyclopedia of Philosophy*, Fall 2016. URL `http://plato.stanford.edu/archives/fall2016/entries/truthmakers/`.

Penelope Maddy. *The Logical Must: Wittgenstein on Logic*. Oxford University Press, 2014.

Elliott Mendelson. *Introduction to Mathematical Logic*. Wadsworth, Inc., third edition, 1987.

Elliott Mendelson. *Introduction to Mathematical Logic*. Chapman & Hall, fourth edition, 1997.

Christopher Menzel. Possible Worlds. *Stanford Encyclopedia of Philosophy*, 2017. URL `https://plato.stanford.edu/archives/win2017/entries/possible-worlds/`.

Kevin Mulligan, Peter Simons, and Barry Smith. Truth-Makers. *Philosophy and Phenomenological Research*, 44(3):287–321, March 1984. URL http://www.jstor.org/stable/2107686.

Daniel Nolan. The Extent of Metaphysical Necessity. *Philosophical Perspectives*, 25(Metaphysics):313–339, 2011. URL https://www.jstor.org/stable/41329474.

Ted Parent. Modal Metaphysics. *Internet Encyclopedia of Philosophy*, 2012. ISSN 2161-0002. URL https://www.iep.utm.edu/mod-meta/.

Katarina Perovic. Can We Be Positive About Russell's Negative Facts? In Landon D. C. Elkind and Gregory Landini, editors, *The Philosophy of Logical Atomism: A Centenary Reappraisal*, History of Analytical Philosophy, chapter 9, pages 199–218. Palgrave MacMillan, London, 2018.

W. V. Quine. Quantification and the Empty Domain. *The Journal of Symbolic Logic*, 19(3):177–179, September 1954.

Greg Restall. Truth-Makers, Entailment and Necessity. In E. J. Lowe and A. Rami, editors, *Truth and Truth-Making*, chapter 4, pages 87–97. McGill-Queen's University Press, 2008. URL https://doi.org/10.1017/UPO9781844654154.005.

Gonzalo Rodriguez-Pereyra. Truthmakers. *Philosophy Compass*, 1(2):186–200, February 2006. doi: 10.1111/j.1747-9991.2006.00018.x.

Bertrand Russell. *Theory of Knowledge: The 1913 Manuscript*, volume 7 of *The Collected Papers of Bertrand Russell*. George Allen and Unwin Ltd., Winchester, Mass., 1913/1983.

Bertrand Russell. The Philosophy of Logical Atomism. In John G. Slater, editor, *The Philosophy of Logical Atomism and Other Essays, 1914-19*, volume 8 of *The Collected Papers of Bertrand Russell*, chapter 17, pages 155–244. George Allen and Unwin Ltd., New York, 1918/1986.

Bertrand Russell. *Introduction to Mathematical Philosophy*. George Allen and Unwin Ltd., 1919.

Bertrand Russell. *The Principles of Mathematics*. George Allen and Unwin Ltd., 2nd edition, 1937.

Bertrand Russell. *An Inquiry into Meaning and Truth*. Penguin University Books, 1940/1973.

Sydney Shoemaker. Causality and Properties. In D. H. Mellor and Alex Oliver, editors, *Properties*, chapter XV, pages 228–254. Oxford University Press, 1997. Reprinted from Peter van Inwagen (1980) *Time and Cause*.

Peter Simons. Logical Atomism and its Ontological Refinement: A Defense. In Kevin Mulligan, editor, *Language, Truth, and Ontology*, Philosophical Studies, pages 157–179. Kluwer, 1992.

Graham Stevens. *The Theory of Descriptions*. History of Analytic Philosophy. Palgrave MacMillan, 2011.

Paul Tomassi. *Logic*. Routledge, 1999.

Donald C. Williams. On the Elements of Being: I. In D. H. Mellor and Alex Oliver, editors, *Properties*, chapter IX, pages 112–124. Oxford University Press, 1997. Reprinted from (1953) *The Review of Metaphysics* Volume 7.

Timothy Williamson. A Note on Truth, Satisfaction, and the Empty Domain. *Analysis*, 59(1):3–8, 1999. URL http://www.jstor.org/stable/3328309.

Timothy Williamson. *Modal Logic as Metaphysics*. Oxford University Press, Oxford, 2013.

Ludwig Wittgenstein. *Notes on Logic*, chapter Appendix B, pages 276–295. Oxford University Press, Oxford, 1913/2009. October 1913.

Ludwig Wittgenstein. *Tractatus Logico-Philosophicus*. International Library of Philosophy and Scientific Method. Routledge & Kegan Paul, second edition, 1922/1971.